Noise of Negative Resistance Oscillators at High Modulation Frequencies

BURKHARD SCHIEK AND KLAUS SCHÜNEMANN

Abstract—The theory of AM and FM noise in oscillators of the negative resistance type is extended to higher modulation frequencies, i.e., to modulation frequencies beyond the bandwidth of the stabilizing cavity. The results are expressed in terms of the input impedance of the stabilizing cavity and are found to agree well with measurements.

I. Introduction

The theory of oscillators, which allows to investigate the influence of the stabilizing circuit on the noise performance of oscillators once the origin of the noise is known, is well established [1]–[7]. The existing theories, however, are limited to small modulation frequencies with respect to the bandwidth of the cavity, as they mostly use Taylor series expansions for the cavity impedance around the frequency of operation. Thus for a high-Q cavity and oscillators operating in the X band the validity of existing theories is limited to modulation frequencies of about 1 MHz. With the advent of superconducting cavities, the 3-dB bandwidth may be of the order of a few hertz, and then the validity range is even more restricted. It is the purpose of this paper to present a noise theory which may be applied to much higher modulation frequencies. The noise theory will be linear, i.e., phase deviations due to noise within the oscillator are assumed to be very small. Furthermore, it is required that the cavity can be described by an equivalent circuit whose input impedance is a quotient of two polynomials in $j\Omega$ with real coefficients, where Ω is the angular frequency. Thus the theory is valid for lumped RLC equivalent circuits with positive or negative RLC's. In an example it will be shown, however, how the theory can be extended if this condition is not satisfied. The presented theory is applicable to negative-resistance-type oscillators such as tunnel diodes, Gunn, and IMPATT diodes, but an extension to an amplifier-type oscillator is possible.

II. General Theory

The theory is based upon the general circuit of Fig. 1. The diode reactance jX_d is assumed to be nearly constant in the small frequency range of interest and to be compensated by, for example, a fictitious and passive inductive reactance $j\Omega L$ in series to the diode reactance. This reactance ($j\Omega L = jX_0$; $X_0 + X_d \simeq 0$) is also nearly

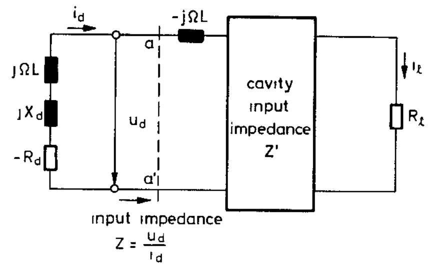

Fig. 1. Equivalent circuit for the oscillator.

constant in the frequency range of interest. The cavity is characterized by its input impedance (Fig. 1)

$$Z'(j\Omega) = \frac{E'(j\Omega)}{F'(j\Omega)}. \tag{1a}$$

It is assumed, and this is very important for the derivation given below, that E' and F' are polynomials in $(j\Omega)$ with real coefficients. Then the impedance in the plane a–a' in Fig. 1 is

$$Z(j\Omega) = -j\Omega L + Z'(j\Omega) = \frac{E(j\Omega)}{F(j\Omega)}. \tag{1b}$$

Also, E and F are polynomials in $(j\Omega)$ with real coefficients.

The current $i_d(t)$ into the impedance Z is not necessarily the same current as the load current i_l. In this case i_d and i_l are related by the current transformation factor $H(j\Omega)$

$$i_d = H(j\Omega)i_l = \frac{N(j\Omega)}{D(j\Omega)} \cdot i_l. \tag{2}$$

Also, the numerator and denominator of $H(j\Omega)$ must be polynomials in $j\Omega$.

The current i_d shows amplitude and phase fluctuations ΔA_d and $\Delta\phi_d$. In the following only a linear noise theory will be derived, i.e., $\Delta A_d/A_0$, $\Delta\phi_d \ll 1$. Then i_d can be expressed as follows:

$$\begin{aligned} i_d(t) &= A_0(t)\cos(\Omega t + \alpha_0 + \Delta\phi_d(t)) \\ &\cong (A_0 + \Delta A_d(t))[\cos(\Omega t + \alpha_0) - \Delta\phi_d(t)\cdot\sin(\Omega t + \alpha_0)] \\ &= \mathrm{Re}\left[A_0 e^{j(\Omega t + \alpha_0) + \Delta\tilde{\phi}_d(t)}\right] \end{aligned} \tag{3}$$

where

$$\left[\frac{\Delta A_d(t)}{A_0} + \Delta\phi_d(t) j\right]$$

has been abbreviated by $\Delta\tilde{\phi}_d(t)$. The fluctuation signals

Manuscript received August 9, 1971; revised May 1, 1972.
B. Schiek is with Philips Forschungslaboratorium Hamburg GmbH, Hamburg 54, Germany, Vogt-Kölln-Str. 30.
K. Schünemann is with Volvo GmbH, Rohren- und Halbleiterwerke, Hamburg 54, Germany, Stresemannallee 101.

$\Delta A_l(t)$, $\Delta\phi_l(t)$, and $\Delta\tilde{\phi}_l(t)$ of the load current $i_l=(A_l+\Delta A_l(t))\cos(\Omega t+\Delta\phi_l(t))$ are defined in a similar way. The fluctuation signals of the load current must now be related to the fluctuation signals of the diode current. Equation (2) gives

$$D\cdot i_d = N\cdot i_l \tag{4}$$

where

$$D = d_m(j\Omega)^m + d_{m-1}(j\Omega)^{m-1} + \cdots$$
$$N = a_n(j\Omega)^n + a_{n-1}(j\Omega)^{n-1} + \cdots. \tag{5}$$

In ac circuit theory $(j\Omega)^m$ is equivalent to the mth derivative d^m/dt^m in the time domain. With (3) and a time dependence of the low-frequency signal $\Delta\phi_d(t)$ of

$$\Delta\phi_d(t) = \tfrac{1}{2}[\Delta\phi_d\cdot e^{j\omega t} + \Delta\phi_d^* e^{-j\omega t}] \tag{6}$$

and similarly for $\Delta\phi_l$, ΔA_d, and ΔA_l, one obtains for the mth derivative of i_d

$$\frac{d^m i_d}{dt^m} = A_0\cdot\frac{d^m}{dt^m}\cdot \text{Re}\left\{e^{j(\Omega t+\alpha_0)} + \left(\frac{\Delta A_d}{A_0}+\Delta\phi_d\cdot j\right)e^{j(\Omega t+\alpha_0)}\cdot\tfrac{1}{2}e^{+j\omega t} + \left(\frac{\Delta A_d^*}{A_0}+\Delta\phi_d^*\cdot j\right)e^{j(\Omega t+\alpha_0)}\cdot\tfrac{1}{2}e^{-j\omega t}\right\} \tag{7a}$$

$$= A_0\cdot\text{Re}\left\{(j\Omega)^m\cdot e^{j(\Omega t+\alpha_0)} + \left(\frac{\Delta A_d}{A_0}+\Delta\phi_d\cdot j\right)\cdot(j\Omega+j\omega)^m\cdot\tfrac{1}{2}e^{j(\Omega t+\alpha_0)}\cdot e^{j\omega t} + \left(\frac{\Delta A_d^*}{A_0}+\Delta\phi_d^*\cdot j\right)\cdot(j\Omega-j\omega)^m\cdot\tfrac{1}{2}e^{j(\Omega t+\alpha_0)}\cdot e^{-j\omega t}\right\}. \tag{7b}$$

The asterisk denotes the complex conjugate.

Introducting (7) into (4) and cancelling the stationary solution yields a relation between ΔA_d, $\Delta\phi_d$, and ΔA_l, $\Delta\phi_l$.

$$|H(j\Omega)|\cdot\sum_m d_m\cdot(j\Omega+j\omega)^m\cdot\left[\frac{\Delta A_d}{A_0}+\Delta\phi_d\cdot j\right]\cdot e^{j\omega t}\cdot e^{j(\Omega t+\alpha_0)} = \sum_n a_n\cdot(j\Omega+j\omega)^n\cdot\left[\frac{\Delta A_l}{A_l}+\Delta\phi_l\cdot j\right]e^{j\omega t}\cdot e^{j\Omega t}. \tag{8}$$

A similar second equation with $j\Omega-j\omega$ is omitted, because its evaluation will give no new results. The further evaluation of (8), which is sketched in the Appendix, yields after some manipulations:

$$\frac{\Delta A_d}{A_0} = \frac{H_1H_0^*+H_0H_{-1}^*}{2H_0H_0^*}\,\frac{\Delta A_l}{A_l} + j\,\frac{H_1H_0^*-H_0\cdot H_{-1}^*}{2H_0H_0^*}\,\Delta\phi_l$$

$$\Delta\phi_d = -j\,\frac{H_1H_0^*-H_0H_{-1}^*}{2H_0H_0^*}\,\frac{\Delta A_l}{A_l} + \frac{H_1H_0^*+H_0H_{-1}^*}{2H_0H_0^*}\,\Delta\phi_l. \tag{9}$$

In (9) the following definitions have been used:

$$H_0 = H(j\Omega)\quad H_1 = H(j\Omega+j\omega)\quad H_{-1} = H(j\Omega-j\omega). \tag{10}$$

Equation (9) may also be used to relate the fluctuation signals ΔU_d, $\Delta\psi_d$ of the diode voltage u_d with ΔA_d, $\Delta\phi_d$ of the diode current i_d if the current transformation factor H is substituted by the input impedance Z. At the center frequency $Z(j\Omega)=Z_0$ (Z_0 real), and $Z_0=-R_d$ (Fig. 1) and one obtains:

$$\frac{\Delta U_d}{U_0} = \frac{Z_1+Z_{-1}^*}{2Z_0}\,\frac{\Delta A_d}{A_0} + j\,\frac{Z_1-Z_{-1}^*}{2Z_0}\,\Delta\phi_d$$

$$\Delta\psi_d = -j\,\frac{Z_1-Z_{-1}^*}{2Z_0}\,\frac{\Delta A_d}{A_0} + \frac{Z_1+Z_{-1}^*}{2Z_0}\,\Delta\phi_d. \tag{11}$$

Z_1 and Z_{-1} are defined as H_1 and H_{-1} in (9). The further development of the theory proceeds in a similar manner as in [2], [3]. With $e(t)$ as the noise source, the diode loop equation for the equivalent circuit of Fig. 1 is

$$i_d(-R_d + j(X_0+X_d)) + u_d = e \tag{12}$$

or

$$\text{Re}\left\{[-R_d + j(X_0+X_d)]A_0e^{j\Omega t+\Delta\tilde{\varphi}_d(t)} + U_0e^{j\Omega t+\Delta\tilde{\phi}_d(t)}e^{\Delta\tilde{\psi}_d(t)-\Delta\tilde{\phi}_d(t)}\right\} = e(t). \tag{13}$$

In the linear noise theory

$$e^{\Delta\tilde{\psi}_d(t)-\Delta\tilde{\phi}_d(t)} = 1 + \Delta\tilde{\psi}_d(t) - \Delta\tilde{\phi}_d(t). \tag{14}$$

Multiplying (13) by $\cos(\Omega t+\Delta\phi_d(t))$ or $\sin(\Omega t+\Delta\phi_d(t))$, integrating over one period of the carrier frequency, and introducing the abbreviations [2]

$$n_1(t) = \frac{2}{T_0}\int_{t-T_0}^{t} e(t')\sin[\Omega t' + \Delta\phi_d(t')]\,dt' \tag{15a}$$

$$n_2(t) = \frac{2}{T_0}\int_{t-T_0}^{t} e(t')\cos[\Omega t' + \Delta\phi_d(t')]\,dt' \tag{15b}$$

$$Z_0 - R_d = \frac{\partial(Z_0-R_d)}{\partial A_0}\Delta A_d = s\cdot\Delta A_d$$

$$X_0 + X_d = \frac{\partial(X_0+X_d)}{\partial A_0}\cdot\Delta A_d = r\cdot\Delta A_d \tag{15c}$$

one finally obtains

$$s\Delta A_d + \left(\frac{Z_1+Z_{-1}^*}{2} - Z_0\right)\frac{\Delta A_d}{A_0} + j\,\frac{Z_1-Z_{-1}^*}{2}\,\Delta\phi_d = \frac{n_2}{A_0} \tag{16a}$$

$$-r\Delta A_d - \left(\frac{Z_1 + Z_{-1}^*}{2} - Z_0\right)\Delta\phi_d + j\frac{Z_1 - Z_{-1}^*}{2}\,\frac{\Delta A_d}{A_0} = \frac{n_1}{A_0}. \quad (16b)$$

Equations (16a) and (16b) describe the AM and FM noise of the diode current i_d as a function of the modulation frequency ω. In the limit of small ω (16a) and (16b) are identical with (4) and (5) of Kurokawa [3] as

$$\lim_{\omega\to 0}\left(\frac{Z_1 + Z_{-1}^*}{2} - Z_0\right) = j\frac{\partial \operatorname{Im} Z}{\partial\omega}\omega$$

and

$$\lim_{\omega\to 0}\frac{Z_1 - Z_{-1}^*}{2} = \frac{\partial \operatorname{Re} Z}{\partial\omega}\omega \quad (17)$$

and also

$$\frac{\partial\Delta A(t)}{dt} = j\omega\Delta A \qquad \frac{\partial\Delta\phi(t)}{\partial t} = j\omega\Delta\phi.$$

Inserting (9) into (16) allows ΔA_l and $\Delta\phi_l$ to be calculated for many practical situations. This will now be illustrated by several examples.

III. Examples of Oscillator Circuits

Equations (9) and (16) can be considerably simplified if one takes the cavity for the oscillator to be symmetrical. Then

$$Z_1 = Z_{-1}^* \quad \text{and} \quad H_1 = H_{-1}^*, H_0 = H_0^* \quad (18)$$

and (9) and (16) can be combined to

$$\frac{\Delta A_l}{A_l} = \frac{H_0 n_2}{H_1(sA_0 + Z_1 - Z_0)A_0} \quad (19a)$$

$$\Delta\phi_l = -\frac{H_0 n_1}{H_1 A_0(Z_1 - Z_0)} - \frac{H_0 r n_2}{H_1(sA_0 + Z_1 - Z_0)(Z_1 - Z_0)}. \quad (19b)$$

Thus a $r \neq 0$ produces a correlation between AM and FM noise.

1) Let us consider the simple RLC-series circuit of Fig. 2, which may represent a transmission-type cavity. With the approximation $j(\Omega+\omega)L + 1/(j(\Omega+\omega)C) \simeq j2\omega L$, which is valid up to at least $\omega/\Omega = 0.05$, and with $H_1 = H_0$ the results are

$$\Delta A_l = \frac{n_2}{sA_0 + j2\omega L} \quad (20a)$$

(transmission cavity, series tuned circuit)

$$\Delta\phi_l = \frac{-n_1}{A_0 2j\omega L} - \frac{r n_2}{(sA_0 + j2\omega L)2j\omega L}. \quad (20b)$$

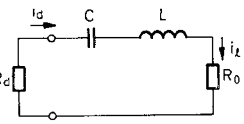

Fig. 2. Transmission cavity represented by a series tuned circuit.

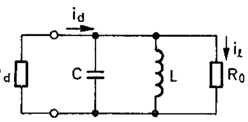

Fig. 3. Transmission cavity represented by a paralled tuned circuit.

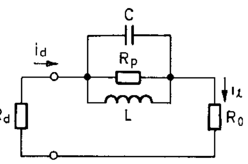

Fig. 4. Equivalent circuit for a reaction-type cavity.

For $r = 0$, this is identical with the results derived in [2]. Contrary to the derivation in [2], the applied approximation for the reactance is convenient but not necessary.

The equivalent circuit of Fig. 3, which is the dual circuit of Fig. 2, may also describe a transmission cavity. With $H_1 = 1 + 2j\omega R_0 C$ and evaluating (19) shows that the asymptotic AM and FM noise behavior is the same as for the series tuned circuit.

2) A possible form of an equivalent circuit for a reaction-type cavity is shown in Fig. 4. The losses of the cavity are determined by R_P, R_0 is the load resitance. As

$$H_1 = H_0$$

$$Z_1 = R_0 + \frac{R_P}{1 + 2j\omega C R_P} \quad (21)$$

one obtains

$$\Delta A_l = \frac{n_2(1 + 2j\omega C R_P)}{sA_0(1 + 2j\omega C R_P) - 2j\omega C R_P^2} \quad (22a)$$

(reaction-type cavity)

$$\Delta\phi_l = \frac{n_1(1 + 2j\omega C R_P)}{A_0 \cdot 2j\omega C R_P^2} + \frac{r n_2(1 + 2j\omega C R_P)^2}{2j\omega C R_P^2[sA_0(1 + 2j\omega C R_P) - 2j\omega C R_P^2]}. \quad (22b)$$

The theoretical results for the transmission and reaction-type cavity—with the assumption of $r \ll s$, i.e., an omission of the second term on the right-hand side of (20b) and (22b)—are shown in Fig. 5(a), (b). In this figure, experimental results obtained with Gunn oscillators at 16 GHz are also presented.

For the theoretical curves in Fig. 5(a), (b), both n_1 and n_2 were assumed to be frequency independent,

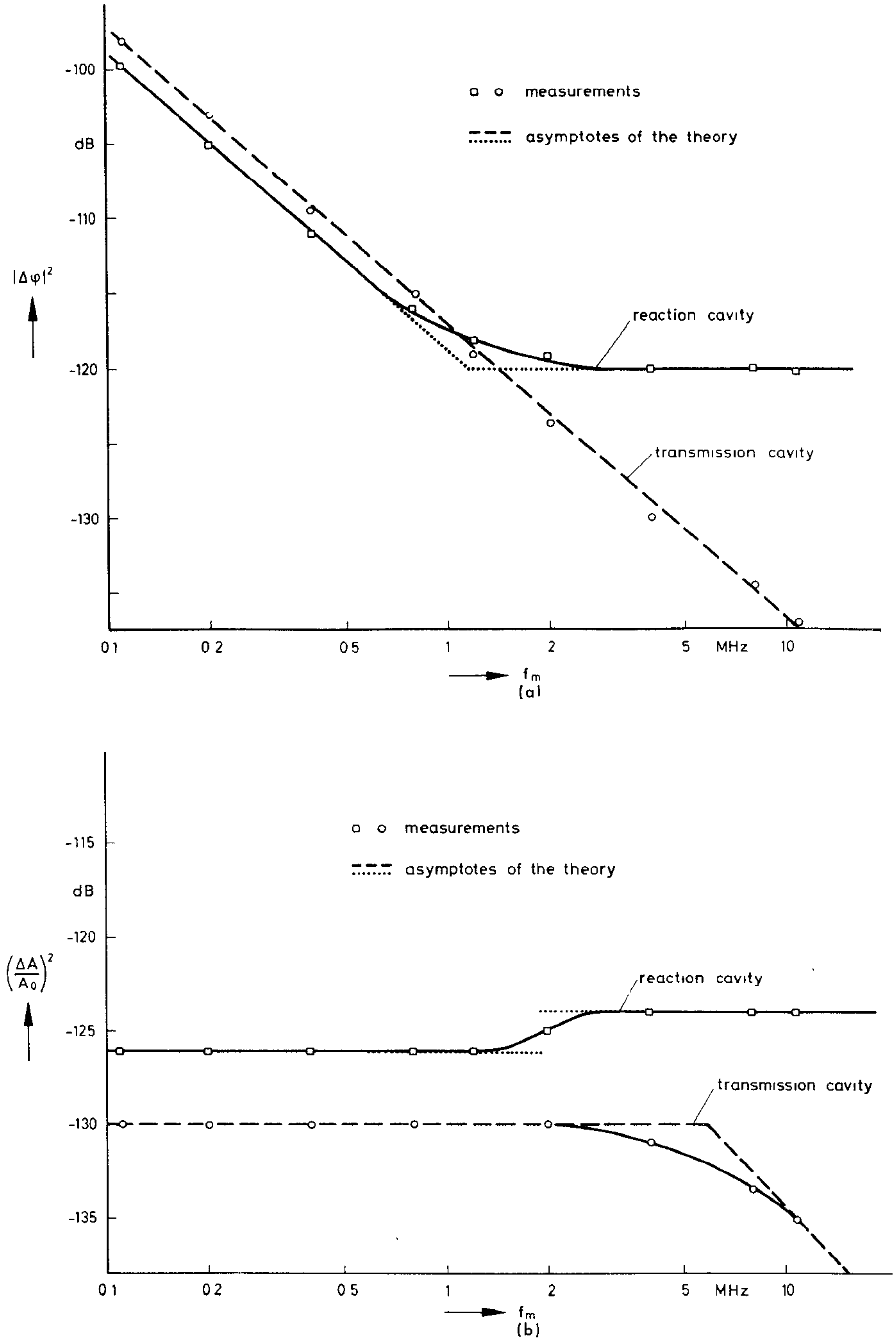

Fig. 5. (a) FM noise spectra of cavity stabilized Gunn oscillators ($f_0 = 16$ GHz, bandwidth 500 Hz). (b) AM noise spectra of cavity stabilized Gunn oscillators ($f_0 = 16$ GHz, bandwidth 500 Hz). $P_{out} \approx 100$ mW.

which is approximately true for Gunn oscillators at higher modulation frequencies. The absolute values for the theoretical curves have been chosen as to coincide to the measured curves at low and high modulation frequencies. The agreement between theory and experiment is good, as can be seen from Fig. 5(a), (b). A detailed discussion of these results has already been given in [8].

3) Let us now assume that the cavity, which is again described by its symmetric input impedance $Z_1 = Z_{-1}^*$, is separated from the diode by a lossless transmission line of electrical length βl (Fig. 6). The frequency dependence of βl is assumed to be negligible with respect to the frequency dependence of Z_1, because the cavity has a high quality factor.

Equation (16) cannot be applied directly to this problem, because the input impedance of a transmission line section terminated by an arbitrary impedance cannot be described by a quotient of polynomials with real coefficients, and therefore the derivation which leads to (16) is no longer valid. In this case i_d and u_d are related to i_z and u_z (Fig. 6) via the transmission line equations.

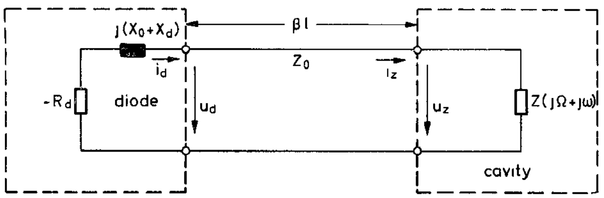

Fig. 6. Diode and cavity separated by a lossless transmission line of electrical length βl.

The fluctuation signals of i_z are defined by:

$$i_z = (A_0 + \Delta A_z) \cos (\Omega t + \Delta\phi_z). \tag{23}$$

Similar definitions hold for the fluctuation signals ΔA_d, $\Delta\phi_d$ of i_d and ΔU_d, $\Delta\psi_d$ of u_d.

Inserting (23) into the transmission line equations, comparing the coefficients of the cos Ωt and sin Ωt terms and using (11) yields linear equations which relate the fluctuation signals:

$$\frac{\Delta A_d}{A_0} = \left[\frac{Z_1 + Z_0}{2Z_0} - \frac{Z_1 - Z_0}{2Z_0} \cos 2\beta l\right]\frac{\Delta A_z}{A_0} - \frac{Z_1 - Z_0}{2Z_0} \sin (2\beta l)\cdot\Delta\phi_z \tag{24}$$

and three similar equations for $\Delta\phi_d$, $\Delta U_d/U_0$, and $\Delta\psi_d$. The evaluation of the diode loop equation yields

$$s\Delta A_d + Z_0\left(\frac{\Delta U_d}{U_0} - \frac{\Delta A_d}{A_0}\right) = \frac{n_2}{A_0} \tag{25a}$$

$$-r\Delta A_d - Z_0(\Delta\psi_d - \Delta\phi_d) = \frac{n_1}{A_0}\cdot \tag{25b}$$

Substituting ΔA_d, ΔU_d, $\Delta\phi_d$, and $\Delta\psi_d$ by (24), one obtains two linear equations which can be solved for ΔA_z and $\Delta\phi_z$:

$$\Delta A_z = \left\{n_2\left[\frac{rA_0}{2Z_0} \sin 2\beta l - \cos 2\beta l\right] + n_1\left(\frac{sA_0}{2Z_0} - 1\right)\sin 2\beta l\right\}\cdot\frac{1}{CoeDe} \tag{26a}$$

$$\Delta\phi_z = \left\{n_1\left[\left(1 - \frac{sA_0}{2Z_0}\right)\sin 2\beta l + \frac{sA_0}{2Z_0}\,\frac{Z_1 + Z_0}{Z_1 - Z_0}\right] + n_2\left[\frac{rA_0}{2Z_0}\left(\frac{Z_1 + Z_0}{Z_1 - Z_0} - \cos 2\beta l\right) - \sin 2\beta l\right]\right\}\cdot\frac{1}{A_0 CoeDe} \tag{26b}$$

$$CoeDe = -(Z_1 - Z_0)\left(1 - \frac{sA_0}{2Z_0}\right) - \frac{Z_1 + Z_0}{2Z_0}\cdot\left[sA_0 \cos 2\beta l - rA_0 \sin 2\beta l\right]. \tag{26c}$$

A considerable simplification of these equations is possible for the condition of maximum output power [2]:

$$\frac{sA_0}{2Z_0} = 1. \tag{27}$$

Then

$$\Delta A_z = \frac{n_2}{Z_1 + Z_0} \tag{28a}$$

$$\Delta\phi_z = \frac{n_1 \cos\theta + n_2 \sin\theta - \dfrac{Z_1 - Z_0}{Z_1 + Z_0}\sin (2\beta l + \theta) n_2}{-(Z_1 - Z_0)\cos (2\beta l + \theta) A_0} \tag{28b}$$

where a diode angle θ has been defined as [3]

$$s = k\cos\theta$$
$$r = k\sin\theta. \tag{29}$$

A simplification is also possible if one considers only low modulation frequencies, i.e., $Z_1 - Z_0 \ll Z_1 + Z_0 \approx 2Z_0$. Then

$$\Delta A_z = \frac{n_2}{2Z_0} + \left(1 - \frac{sA_0}{2Z_0}\right)\frac{n_2 \cos 2\beta l + n_1 \sin 2\beta l}{kA_0 \cos (2\beta l + \theta)} \tag{30a}$$

$$\Delta\phi_z = \frac{n_1 \cos\theta + n_2 \sin\theta}{-(Z_1 - Z_0)\cos (2\beta l + \theta) A_0}\cdot \tag{30b}$$

Thus, for both approximations, the AM noise is nearly independent of the line length βl, while the FM noise power varies essentially proportional to $\cos^{-2}(2\beta l + \theta)$. For low modulation frequencies this result may be shown to be valid for a general circuit which satisfies the following two conditions. 1) The transforming network between the diode and the load is lossless but otherwise arbitrary; losses in series or parallel to both the load and the negative resistance are allowed. 2) The oscillator is tuned to maximum output power [See (27)].

For small modulation frequencies and neglecting the very small AM–FM conversion (9) reads

$$\lim_{\omega\to 0}\begin{cases}\dfrac{\Delta A_d}{A_d} = \dfrac{\Delta A_l}{A_l} + j\omega\,\dfrac{\partial(HH^*)}{\partial\omega}\cdot\dfrac{1}{2HH^*}\,\Delta\phi_l\\ \Delta\phi_d = \Delta\phi_l.\end{cases} \tag{31}$$

The condition that the transforming network is lossless relates Re Z and H via

$$\frac{\partial\,\mathrm{Re}\,(Z)}{\partial\omega} + Z_0\,\frac{\partial(HH^*)}{\partial\omega}\,\frac{1}{HH^*} = 0. \tag{32}$$

Inserting (27), (31), and (32) into (16) or its approximate form (17) yields for the AM fluctuations of the load current

$$\frac{\Delta A_l}{A_l} = \frac{n_2}{A_0(s\cdot A_0)} \tag{33a}$$

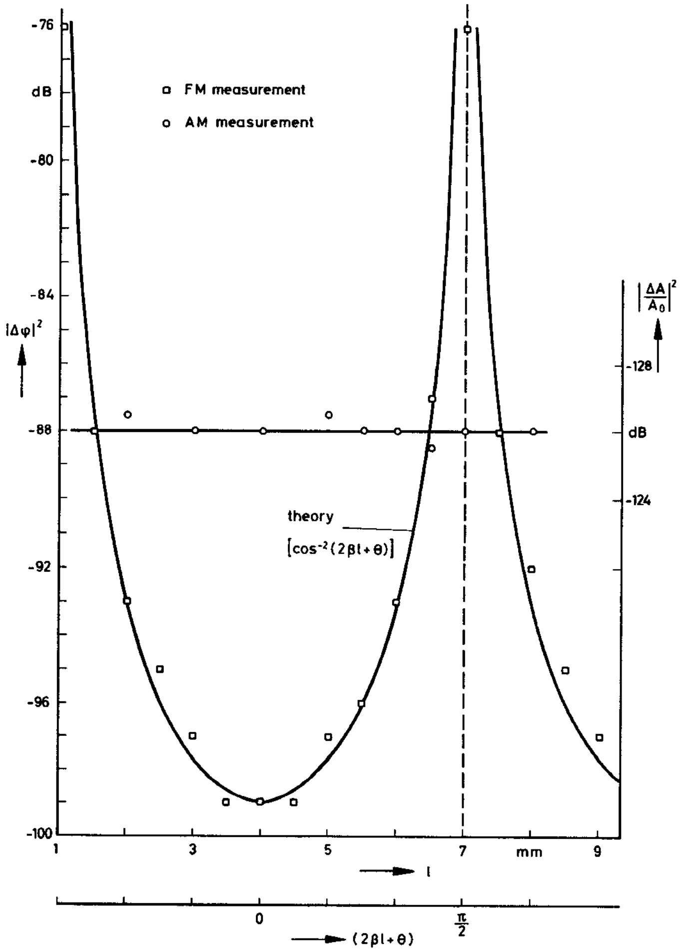

Fig. 7. Noise of a reaction cavity stabilized Gunn oscillator versus the distance of the cavity from the oscillator ($P_{out}=115$ mW, $f_0=16$ GHz, at 100 kHz from the carrier in 5-kHz bandwidth).

while the FM fluctuations can be calculated from (16) and (17) to be

$$i\omega A_0 \cdot \Delta\phi_l = \frac{n_1 \cdot \cos\theta + n_2 \cdot \sin\theta}{-\dfrac{\partial \operatorname{Im}(Z)}{\partial\omega} \cdot \cos\theta + \dfrac{\partial \operatorname{Re} Z}{\partial\omega} \sin\theta} \cdot \qquad (33b)$$

Equations (33a) and (33b) are very similar to (30a) and (30b).

Thus the AM noise is independent of the particular tuning of the outer circuit at low modulation frequencies under rather general conditions. The results obtained have been verified by measurements which have been performed on the same cavity stabilized oscillators as in Fig. 5. The results for the reaction cavity are shown in Fig. 7. Care has been taken in order to tune the oscillator for maximum output power. The distance between the oscillator and the cavity has been varied by a set of disks. The measurements agree well with the theoretical predictions as is also true for the transmission cavity.

IV. Conclusion

The linear noise theory of oscillators of the negative resistance type has been extended to high modulation frequencies, i.e., to modulation frequencies which are higher than the bandwidth of the stabilizing cavity. Although the formulas obtained are very simple to use they are only applicable to lumped equivalent RLC networks with positive or negative R, L, C elements. By way of an example, it is shown how the theory can also be extended to combinations of lumped equivalent circuits and transmission line sections. Furthermore, the theory has been applied to equivalent circuits of transmission and reaction-type cavities and a close agreement between theory and measurements has been found. For low modulation frequencies it is shown that the AM noise is independent of the particular tuning of the circuit under rather general conditions.

Appendix

When evaluating (8) one has to bear in mind that the operator $j\Omega$ in (8) only applies to the carrier $e^{j\Omega t}$, while the operator $j\omega$ applies to the modulation signal $e^{j\omega t}$. This means that $j\Omega$ changes the phase of the carrier by 90°, while $j\omega$ changes the phase of the modulation signal. Comparing the coefficients of $\cos \Omega t$ and $\sin \Omega t$ of (8) yields two equations which relate the fluctuation signals of i_d and i_l.

In the equation below $(\operatorname{Re} D)_e$ is the even part of the real part of D with respect to ω, $(\operatorname{Re} D)_o$ the odd part, $(\operatorname{Im} D)_{e,o}$ are the corresponding imaginary parts of D, respectively, etc. Writing, similar to (10), $D_1 = D(j\Omega + j\omega)$, $D_{-1}^* = D^*(j\Omega - j\omega)$ and

$$\tfrac{1}{2}(D_1 + D_{-1}^*) = (\operatorname{Re} D)_e + j(\operatorname{Im} D)_o$$
$$\tfrac{1}{2}(D_1 - D_{-1}^*) = (\operatorname{Re} D)_o + j(\operatorname{Im} D)_e \qquad (34)$$

and similarly for N, (8) becomes:

$$|H_0| \left\{(D_1 + D_{-1}^*) \frac{\Delta A_d}{A_0} + j(D_1 - D_{-1}^*) \cdot \Delta\phi_d\right\} \cos\alpha_0$$
$$+ |H_0| \cdot \left\{-(D_1 + D_{-1}^*)\Delta\phi_d + j(D_1 - D_{-1}^*) \frac{\Delta A_d}{A_0}\right\}$$
$$\cdot \sin\alpha_0 = (N_1 + N_{-1}^*) \cdot \frac{\Delta A_l}{A_l} + j(N_1 - N_{-1}^*) \cdot \Delta\phi_l \qquad (35)$$

and a similar second equation.

Solving (35) for ΔA_d and $\Delta\phi_d$ and using

$$A_0 \cdot \cos\alpha_0 = \frac{H_0 + H_0^*}{2} \cdot A_l$$
$$A_0 \sin\alpha_0 = \frac{H_0 - H_0^*}{2j} A_l \qquad (36)$$

and

$$\frac{N_1 D_{-1}^* + N_{-1}^* D_1}{D_1 \cdot D_{-1}^*} = H_1 + H_{-1}^* \qquad (37)$$

and a similar equation for $H_1 - H_{-1}^*$, yields (9) of Section II. Equation (9) has already been obtained by Tellegen and van Nie [9] in a somewhat different form as has recently been pointed out to the authors by van Nie.

Acknowledgment

The authors wish to thank J. K. Vogel, W. Schilz, J. Köhler, and J. Magarshack for valuable comments and discussions.

References

[1] W. A. Edson, "Noise in oscillators," *Proc. IRE*, vol. 48, pp. 1454–1466, Aug. 1960.
[2] K. Kurokawa, "Noise in synchronized oscillators," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-16, pp. 234–240, Apr. 1968.
[3] ——, "Some basic characteristics of broadband negative resistance oscillator circuits," *Bell Syst. Tech. J.*, vol. 48, pp. 1937–1955, 1969.
[4] E. J. Shelton, Jr., "Stabilization of microwave oscillators," *IRE Trans. Electron Devices* (*Symposium Issue*), vol. ED-1, pp. 30–40, Dec. 1954.
[5] I. Goldstein, "Frequency stabilization of a microwave oscillator with an external cavity," *IRE Trans. Microwave Theory Tech.*, vol. MTT-5, pp. 57–62, Jan. 1957.
[6] J. R. Ashley and C. B. Searles, "Microwave oscillator noise reduction by a transmission stabilizing cavity," *IEEE Trans. Microwave Theory Tech.* (*Special Issue on Noise*), MTT-16, pp. 743–748, 1968.
[7] C. Müller, "Unterdrückung des Frequenz-Rauschens von Gunn-Oszillatoren mit Hilfe eines äusseren Resonators," *Frequenz*, vol. 23, pp. 364–368, 1969.
[8] K. Schünemann and B. Schiek, "A comparison between transmission and reaction cavity stabilized oscillators," *Electron. Lett.*, vol. 7, p. 618, Oct. 1971.
[9] B. D. H. Tellegen and A. G. van Nie, unpublished work, Eindhoven, The Netherlands, 1957.

Stability Criteria for Phase-Locked Oscillators

G. H. BERTIL HANSSON AND K. INGEMAR LUNDSTRÖM

***Abstract*—Stability criteria for negative conductance oscillators or amplifiers are derived in terms of the total circuit admittance. A figure of merit for phase locking at small injected powers is derived. The influence of large injected signals is studied. The conclusions drawn from the calculations are in good qualitative agreement with experimental observations on phase-locked IMPATT-diode oscillators.**

I. Introduction

PHASE-LOCKED oscillators have been shown a large interest in recent years due to the possibility of decreasing the FM noise of solid-state oscillators by injection locking. The purpose of this paper is to derive some general stability criteria for amplifiers and phase-locked oscillators whose active element can be described as a negative conductance (or negative resistance). The analysis is similar to that used by Kurokawa [1] and Brackett [2], who considered a general circuit in contrast to Adler [3], who studied a simple single resonant circuit. The stability criteria for a phase-locked oscillator are derived in a different way and cast in a different form that we find convenient to use. The main difference is, however, that we use a general series expansion for the negative conductance in contrast with Kurokawa who used a first-order approximation [1, eq. (11)]. One of the results of our theory is the introduction of two border lines for stable locking [4], which are called the *boundary* and *locus* curve, respectively, using a notation introduced by Golay [5], who studied the stability of a regenerative oscillator. It is shown by experiments that these two curves have practical implications. By calculating the boundary and locus curves, hysteresis and jumps in output power can be predicted.

Manuscript received November 22, 1971; revised March 6, 1972.
The authors are with the Research Laboratory of Electronics III, Chalmers University of Technology, Gothenburg, Sweden.

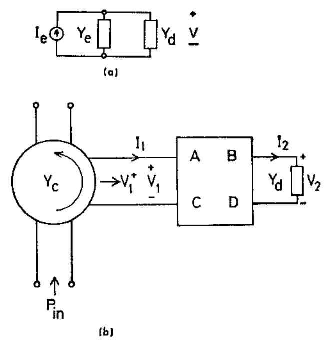

Fig. 1. (a) Equivalent circuit. (b) Circulator coupled negative conductance element.

The theory is applied to a simple cubic nonlinearity, with both a nonlinear conductance and susceptance. It is shown that the nonlinear susceptance introduces asymetrical locking properties at large injected powers.

II. Circuit Equations

The starting point for our calculations is the equivalent circuit shown in Fig. 1(a). In this circuit I_e is a current of frequency ω_i, which depends on the injected power P_{in}. Y_e is the admittance of the passive circuit as seen from the active element. The active element is described by a voltage-dependent susceptance

$$Y_d = G_d(V, \omega) + jB_d(V, \omega) \tag{1}$$

where V is the amplitude of the RF voltage across the active element. Y_e and I_e depend on the actual circuit. A circulator coupled negative conductance element, shown in Fig. 1(b), where the coupling circuit is de-